Multifractality of Brownian motion near absorbing polymers

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We characterize the multifractal behavior of Brownian motion in the vicinity of an absorbing star polymer. We map the problem to an $O(M)$-symmetric ϕ^4-field theory relating higher moments of the Laplacian field of Brownian motion to corresponding composite operators. The resulting spectra of scaling dimensions of these operators display the convexity properties that are necessarily found for multifractal scaling but unusual for power of field operators in field theory. Using a field-theoretic renormalization group approach we obtain the multifractal spectrum for absorption at the core of a polymer star as an asymptotic series. We evaluate these series using resummation techniques. [S1063-651X(99)06306-0]

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I. INTRODUCTION

The concept of multifractality developed in the last decade has proven to be a powerful tool for analyzing systems with complex statistics that otherwise appear to be intractable [1,2]. It has found direct application in a wide range of fields, including turbulence, chaotic attractors, Laplacian growth phenomena, etc. [3,4]. Let us give a simple example of a multifractal (MF) phenomenon. On a possibly fractal set $X \subset \mathbb{R}^d$ of total size R, a field $\varphi(r)$ is given at a microscopic scale ℓ. Then normalized moments of this field may have power law scaling behavior for $\ell/R \to 0$:

$$\langle \varphi(r)^n \rangle / \langle \varphi(r) \rangle^n \sim (R/\ell)^{-\tau_n}. \tag{1}$$

Nontrivial multifractal scaling is found if $\tau_n \neq 0$. When the moments are averages over the sites of X, $\varphi(r)$ defines a measure on X and rigorous arguments show that the τ_n are convex from below as functions of n [5,6].

Here, we generalize an idea of Cates and Witten [7] by deriving the MF spectrum in the frames of a field-theoretical formalism and make use of renormalization group (RG) methods accompanied by the resummation technique. To our knowledge the latter has not been applied before in the theory of multifractals. We relate the MF spectrum to the spectrum of scaling dimensions of a family of composite operators of Lagrangian ϕ^4 field theory. This gives an example of power of field operators whose scaling dimensions show the appropriate convexity for a MF spectrum [6,8–10].

We thus address a special case of a growth process controlled by a Laplacian field. The latter may describe a variety of phenomena, depending on the interpretation of the field. For diffusion limited aggregation (DLA) this field is given by the concentration of diffusing particles, in solidification processes it is given by the temperature field, in dielectric breakdown it is the electric potential, and in viscous finger formation it is the pressure [4]. In all these processes the resulting structure appears to be of a fractal nature and is characterized by appropriate fractal dimensions [12]. The growth and spatial correlations of the structure are governed by spectra of multifractal dimensions [1,2]. In general, the boundary conditions determining the field will be given on the surface of the growing aggregate itself. It is this dynamic coupling that produces the rich structure of the phenomena and seems to make the general dynamical problem intractable analytically. The scaling of the moments of the concentration near the surface of a DLA aggregate has been investigated in detail by Monte Carlo (MC) methods. Simulations with high accuracy show that in $d=3$ dimensions this scaling is described by the MF formalism, while in $d=2$ no power law scaling for higher moments occurs [11].

Here, we study a simpler case where the fractal structure is given and we look for the distribution of a Laplacian field $\rho(r)$ and its higher moments near the surface of the structure [7]. We will follow the diffusion picture, considering the aggregate as an absorbing fractal, ''the absorber.'' The field $\rho(r)$ gives the concentration of diffusing particles and vanishes on the surface of the absorber. More specifically, we consider the Laplacian field $\rho(r)$ in the vicinity of an absorbing polymer, or near the core of a polymer star. In general, we assume the ensemble of absorbers to be characterized by either random walk (RW) or self-avoiding walk (SAW) statistics. Multifractal scaling is found for the n moments $\langle \rho^n(r) \rangle$ of the field with respect to these ensembles. The two dimensional version of this problem is under current discussion. Sets of multifractal exponents for $d=2$ have been proposed recently with exact results from conformal field theory [13] and with perturbative results using the present approach [10].

This formulation of the problem allows us to use the polymer picture and theory developed for polymer networks and stars [14,15] and extended to copolymer stars [9,10]. The theory is mapped to a Lagrangian ϕ^4 field theory with several couplings [16–18] and higher order composite operators [15,9,10] to describe star vertices.

Our paper is organized in the following way. In the next section we present the path integral formulation of the Laplace equation and relate it to a polymer representation. The field-theoretical representation and renormalization of this polymer model aare discussed in Sec. III, where we discuss the renormalization group flow and corresponding expressions for the exponents τ_n. We calculate the multifractal spectrum to third order of perturbation theory using two

complementary approaches: the zero mass renormalization with successive ε expansion (see, e.g., [19]) and the massive renormalization group approach at fixed dimension [20]. For some special cases we reproduce previous results [7] that were obtained in a lower order of perturbation theory. In Sec. IV we derive the multifractal spectra in terms of series expansions. These we present in both of our RG approaches. The resulting series are asymptotic. In Sec. V we take this into account and obtain numerical values only by careful resummation. In Sec. VI we discuss our results and conclude the present study.

II. PATH INTEGRAL SOLUTION OF THE LAPLACE EQUATION AND POLYMER ABSORBER MODEL

In this section we show how to describe the diffusion of particles in the presence of an absorbing polymer using a ''polymer'' formalism that represents both the random walks of the diffusing particles and the absorber itself in the same way [7,21]. Let us formulate this problem first in terms of the diffusion of particles in time. The probability of finding a diffusing particle at point r_1 at time t if it started at point r_0 at time $t=0$ can be described by the following normalized path integral:

$$G^0(r_0,r_1,t)=\langle \delta(r^{(1)}(0)-r_0)\delta(r^{(1)}(t)-r_1)\rangle_{\mathcal{H}_0(r^{(1)},t)}. \tag{2}$$

The angular brackets in Eq. (2) stand for the following average:

$$\langle \cdots \rangle_{\mathcal{H}_0(r^{(1)},t)}=\frac{\int (\cdots)\exp[-\mathcal{H}_0(r^{(1)},t)]d\{r^{(1)}\}}{\int \exp[-\mathcal{H}_0(r^{(1)},t)]d\{r^{(1)}\}}, \tag{3}$$

which is performed with the Hamiltonian

$$\mathcal{H}_0(r^{(1)},t)=\int_0^t \left(\frac{dr^{(1)}(\tau)}{2\,d\tau}\right)^2 d\tau. \tag{4}$$

The integration in Eq. (2) is performed over all paths $r^{(1)}(\tau)$ with $0\leqslant\tau\leqslant t$. Note that we have absorbed the diffusion constant by a redefinition of time. The unit of the dimensionless Hamiltonian $\mathcal{H}_0$ is the product k_BT of Boltzmann's constant and temperature, while that of time t is the square microscopic length ℓ^2. Spatial boundaries may be included in Eq. (2) by restricting the path integral to a subspace. The path integrals in Eq. (3) are Gaussian and may be performed with the boundary conditions of Eq. (2), and are shown to be equivalent to the solution of the following harmonic differential equation [22]:

$$\left(\Delta_{r_0}-\frac{\partial}{\partial t}\right)G^0(r_0,r_1,t)=0,\;\; G^0(r_0,r_1,0)=\delta(r_1-r_0). \tag{5}$$

We now introduce the absorbing polymer into the system volume. The latter is assumed to be of much larger size than the absorber itself. The probability of diffusing from r_0 to r_1 is now $G(r_0,r_1,t)$. Walks that touch the absorber cannot contribute to this probability. The absorber itself we describe by a path $r^{(2)}(s)$, $0\leqslant s\leqslant S_2$. The boundary condition is implemented by an avoidance interaction u_{12} punishing any coincidence of the path $r^{(1)}$ of the RW and the path $r^{(2)}$ of the absorber. The correlation function of a random walk in the presence of an absorbing path $r^{(2)}(s)$ with $0\leqslant s\leqslant S_2$ may then be written as

$$G(r_0,r_1,S_1)=\Bigg\langle \delta(r^{(1)}(0)-r_0)\delta(r^{(1)}(S_1)-r_1) \times\exp\Bigg\{-\frac{u_{12}}{3!}\int_0^{S_1}ds_1\int_0^{S_2}ds_2\,\delta(r^{(1)}(s_1)-r^{(2)}(s_2))\Bigg\}\Bigg\rangle_{\mathcal{H}_0(r^{(1)},S_1)}, \tag{6}$$

where we have adopted the notation $t=S_1$. Obviously this is symmetric in r_0 and r_1. The probability of finding a particle at $t=0$ in r_0 if it was launched at time $t=-\infty$ at any point r_1 in the volume is then described by the field

$$\rho(r_0)=\lim_{S_1\to\infty}\frac{1}{V}\int dr_1 G(r_0,r_1,S_1). \tag{7}$$

Due to Eq. (5), which is not perturbed in the volume outside the absorber, $\rho(r_0)$ obeys the Laplace equation

$$\Delta\rho(r)=0, \tag{8}$$

with the following boundary conditions: Because of the avoidance condition ρ vanishes on the absorber. If the external volume boundary ∂V is far enough from the absorber there is equal probability of reaching any point on ∂V resulting in a constant concentration $\rho=\rho_\infty$ on ∂V.

We are interested in ensemble averaged moments $\langle\rho^n(r_0+\xi)\rangle$ of the field in the vicinity of the absorber, i.e., with microscopic ξ. For the RW ensemble the average is performed with respect to the Hamiltonian $\mathcal{H}_0(r^{(2)},S_2)$; for the SAW ensemble an additional interaction has to be included. The moments we calculate as an ensemble average over all configurations of the absorbing polymer choosing the site r_0 on the middle of the polymer [23]. Formally we write these moments for $\xi\to 0$ as

$$\lim_{|\xi|\to 0}\langle\rho^n(r_0+\xi)\rangle = \lim_{S_{a>m}\to\infty}\frac{1}{\mathcal{Z}^0_{*m0}}\int\prod_{a=1}^{m+n}dr_a \times G^*_{mn}(r_0,r_1,\ldots,r_{m+n},S_1,\ldots,S_{m+n}). \tag{9}$$

The normalization $\mathcal{Z}^0_{*m0}$ takes care of the configurations of the absorber, as explained in the next section. The correlation function G^*_{mn} is defined as

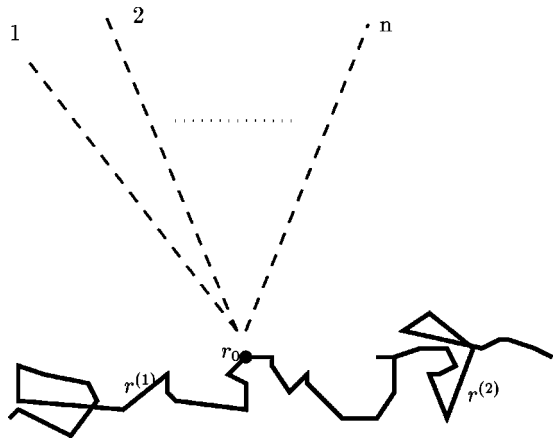

FIG. 1. n random walks that end at point r_0 on the absorbing polymer.

$$G^*_{mn}(r_0,r_1,\ldots,r_{m+n},S_1,\ldots,S_{m+n}) = \left\langle \prod_{a=1}^{m+n} \delta(r^{(a)}(0)-r_0)\delta(r^{(a)}(S_a)-r_a) \times \exp\left\{ -\sum_{a,b=1}^{m+n} \frac{\overline{u}_{ab}}{3!}\int_0^{S_a} ds_a \int_0^{S_b} ds_b \times \delta(r^{(a)}(s_a)-r^{(b)}(s_b)) \right\} \right\rangle_{\sum_a \mathcal{H}_0(r^{(a)},S_a)} . \quad (10)$$

Here, the absorbing walk is represented by $m=2$ paths $r^{(1)}$, $r^{(2)}$, while the remaining n paths represent n random walks, as is shown in Fig. 1. The interaction matrix $\overline{u}_{ab}$ is in this case given by $\overline{u}_{ab}=\{0$ if $a,b\leqslant m$ or $a,b>m$; u_{12} otherwise$\}$.

The limits in Eq. (9) look rather ill defined at first sight, and indeed they should not be taken naively. Also the evaluation of the functional integral (10) is not defined in this bare form. Luckily we have at hand the polymer field theory that has dealt with the problems of evaluating these formal expressions [24]. We will show below how the theory is mapped to a renormalizable $O(M)$ symmetric field theory in terms of which the limits and a perturbative expansion of Eq. (10) make sense. For instance, the limit $|\xi|\to 0$ may be interpreted as a short distance limit defining a composite operator, while the limit $S_{a>m}\to\infty$, with $S_{b\leqslant m}$ staying finite, corresponds to a short chain limit derived in [25]. In the frames of the polymer picture we may interpret $G^*_{mn}(r_0,r_1,\ldots,r_{m+n},S_1,\ldots,S_{m+n})$ as the correlation function of $m+n$ interacting walks all starting at point r_0 with end points at $r_1,\ldots,r_{m+n}$. These describe what is called a polymer star. The normalized partition function of such a star of $m+n$ polymer chains with chain lengths parametrized by S_a may be written as [14,15]

$$\mathcal{Z}_{*mn}\{S_a\} = \frac{1}{\mathcal{N}_{mn}} \int \prod_{a=1}^{m+n} \mathrm{d}r_a \times G^*_{mn}(r_0,r_1,\ldots,r_{m+n},S_1,\ldots,S_{m+n}). \quad (11)$$

The normalization $\mathcal{N}_{mn}$ is chosen such that $\mathcal{Z}_{*mn}\{S_a\}|^{-}_{u_{ab}=0}=1$ for vanishing interactions and the point r_0 is arbitrary. We have studied this problem of polymer stars with general interaction matrix $\overline{u}_{ab}$ in [9,10]. Here we will choose

$$\overline{u}_{ab} = \begin{cases} u_{11} & \text{if } a,b\leqslant m \\ u_{22} & \text{if } a,b>m \\ u_{12} & \text{otherwise.} \end{cases} \quad (12)$$

This allows us to think of the absorbing paths $r^{(1)},\ldots,r^{(m)}$ as being either of a RW ($u_{11}=0$) or of a SAW ($u_{11}\neq 0$) ensemble. With $m>2$ we also include in our study the moments of the diffusion field near to the core of a polymer star. We included u_{22} only to ease notation; in the present context $u_{22}=0$.

III. FIELD THEORY AND RENORMALIZATION

As is well known, the polymer model may be mapped to the limit $M=0$ of $O(M)$-symmetrical Lagrangian field theory [26]. We adopt the formalism developed for multicomponent polymer solutions that allows us to describe both polymers and interacting random walks [18]. Its field theory is given by the following Lagrangian:

$$\mathcal{L}\{\phi_a,\mu_a\} = \frac{1}{2}\sum_{a=1}^{m+n}\int d^dr\{\mu_a\phi_a^2+[\nabla\phi_a(r)]^2\} + \frac{1}{4!}\sum_{a,b=1}^{m+n}\overline{u}_{a,b}\int d^dr\,\phi_a^2(r)\phi_b^2(r). \quad (13)$$

The ϕ^2 terms should be read as scalar products of fields ϕ_a with M component

$$\phi_a^2 = \sum_{\alpha=1}^{M}(\phi_a^\alpha)^2. \quad (14)$$

The parameter μ_a is a chemical potential conjugated to the chain length variables S_a in Eq. (6). Correlation functions in this theory are defined by averaging with the Lagrangian $\mathcal{L}$:

$$\langle(\cdots)\rangle_{\mathcal{L}} = \int \mathcal{D}[\phi_a(r)](\ldots)\exp[-\mathcal{L}\{\phi_a,\mu_a\}]\,|_{M=0}. \quad (15)$$

Here, functional integration $\int\mathcal{D}[\phi_a(r)]$ is defined in such a way that normalization is already included: $\langle 1\rangle|_{\mathcal{L}}=1$ if all $\overline{u}_{a,b}\equiv 0$. The limit $M=0$ in Eq. (15) can be understood as a selection rule for the diagrams that contribute to the perturbation theory expansion and can be easily checked in this context to correspond to interacting polymers in the following way: The partition function $\mathcal{Z}_{*mn}$ defined in Eq. (6) is mapped to the field-theoretical correlation function $\tilde{\mathcal{Z}}_{*mn}$ via Laplace transforms in the chain length variables S_a to conjugate chemical potentials (''mass variables'') μ_a:

$$\tilde{\mathcal{Z}}_{*mn}\{\mu_a\} = \int_0^\infty \prod_b dS_b e^{-\mu_b S_b}\mathcal{Z}_{*mn}\{S_a\}. \quad (16)$$

In terms of the above defined Lagrangian field theory, $\tilde{\mathcal{Z}}_{*mn}$ is given by

$$\tilde{\mathcal{Z}}_{*mn}\{\mu_a\}=\left\langle \int \prod_{a=1}^{m+n} dr_a \phi_a(r_0)\phi_a(r_a)\right\rangle\Bigg|_{\mathcal{L}}. \quad (17)$$

Our interest is in the scaling properties of these functions. Note that in Eq. (17) these are governed by the spectrum of scaling dimensions of the composite operators $\Pi_{a=1}^{m+n}\phi_a$. To extract these dimensions we use RG methods [27,28]. Here, we apply the results of our previous approaches to the problem of copolymer stars [9,10]: massless renormalization group scheme with successive ε expansion (see, e.g., [19]) and massive renormalization group approach at fixed dimension [20] compiled in a pseudo-ε-expansion [29]. On the basis of correlation functions it is standard to define vertex functions $\Gamma^{(4)}_{u_{ab}}$ corresponding to the couplings $\bar{u}_{ab}$ as well as vertex functions $\Gamma^{*mn}_{\Pi\phi_a}$ with the insertion of composite operators $\Pi_a\phi_a$. Explicit expressions may be found in [10]. We define renormalization and introduce renormalized couplings $\bar{g}_{ab}$ by

$$\bar{u}_{ab}=\mu^{\varepsilon}Z_{\phi_a}Z_{\phi_b}Z_{ab}\bar{g}_{ab}. \quad (18)$$

The renormalizing Z factors are power series in the renormalized couplings $\bar{g}_{ab}$ subject to the following renormalization conditions:

$$Z_{\phi_a}(\bar{g}_{aa})\frac{\partial}{\partial k^2}\Gamma^{(2)}_{aa}(\bar{u}_{aa}(\bar{g}_{aa}))=1, \quad (19)$$

$$Z_{ab}(\{\bar{g}_{ab}\})\Gamma^{(4)}_{aabb}(\bar{u}_{ab}(\{\bar{g}_{ab}\}))=\mu^{\varepsilon}\bar{g}_{ab}. \quad (20)$$

The scale parameter μ is equal to the mass at which the massive scheme is evaluated or it gives the scale of the external momenta in the massless scheme.

In order to renormalize the star vertex functions we introduce renormalization factors $Z^{*mn}_{\Pi\phi_a}$ by

$$\left(\prod_{a=1}^{k} Z^{1/2}_{\phi_a}\right)Z^{*mn}_{\Pi\phi_a}\Gamma^{*mn}_{\Pi\phi_a}(\bar{u}_{ab}(\{\bar{g}_{ab}\}))=\mu^{\delta_{\Pi\phi_a}}, \quad (21)$$

where $\delta_{\Pi\phi_a}$ is the engineering dimension of the composite operator

$$\delta_{\Pi\phi_a}=(m+n)\left(\frac{\varepsilon}{2}-1\right)+4-\varepsilon. \quad (22)$$

The renormalized couplings $\bar{g}_{ab}$ and renormalizing Z factors depend on the scale parameter μ. This is expressed by the following RG flow equations:

$$\mu\frac{d}{d\mu}\bar{g}_{ab}=\bar{\beta}_{ab}(\{\bar{g}_{ab}\}), \quad (23)$$

$$\mu\frac{d}{d\mu}\ln Z^{*mn}_{\Pi\phi_a}(\{\bar{g}_{ab}\})=\eta_{\Pi\phi_a}(\{\bar{g}_{ab}\}). \quad (24)$$

Our original problem is described by two sets of walks of different species. In this case only the three different couplings u_{11}, u_{12}, and u_{22} in Eq. (12) remain. We will refer to their renormalized counterparts as g_{11}, g_{22}, $g_{12}=g_{21}$. The corresponding functions β_{11}, β_{22}, β_{12} define the flow in the space of couplings. This RG flow was discussed in [17,18]. Its fixed points are determined by the set of equations

$$\beta_{11}(g^*_{11})=0,$$

$$\beta_{22}(g^*_{22})=0, \quad (25)$$

$$\beta_{12}(g^*_{11},g^*_{22},g^*_{12})=0.$$

In the space of the three couplings one finds [18] eight fixed points corresponding to the absence or presence of inter- and intraspecies interaction. The equations for the fixed points of the β functions were found to have the following nontrivial solutions: $\beta_{aa}(g^*_S)=0$; and for $a\neq b$, $\beta_{12}(0,0,g^*_G)=0$, $\beta_{12}(g^*_S,0,g^*_U)=0$, $\beta_{12}(0,g^*_S,g^*_U)=0$, and $\beta_{12}(g^*_S,g^*_S,g^*_S)=0$, corresponding to all combinations of interacting and noninteracting chains.

The phenomenon we address in this paper corresponds to the case of a nonvanishing interaction between the two species of walks, while one set has no self-interaction. Thus we consider the two fixed points, which we call G ($g_{11}=g_{22}=0, g_{12}=g^*_G$) and U ($g_{11}=g^*, g_{22}=0, g_{12}=g^*_U$). The first ($G$) corresponds to a set of random walks interacting with another set of random walks of a second species and thus describes absorption on random walk absorbers; the second (U) corresponds to a set of random walks interacting with a set of self-avoiding walks and thus describes absorption on SAW (polymer) absorbers.

Having m walks of the first species and n walks of second species we define the following exponents in the fixed points G,U:

$$\eta^G_{mn}=\eta_{\Pi\phi_a}(g_{11}=g_{22}=0,g_{12}=g^*_G), \quad (26)$$

$$\eta^U_{mn}=\eta_{\Pi\phi_a}(g_{11}=g^*,g_{22}=0,g_{12}=g^*_U), \quad (27)$$

which govern the scaling properties of the partition sum (11).

The scaling may be formulated in terms of the size R of the absorbing walks, while the RWs of the diffusing particles are taken infinitely long. This corresponds to a short chain expansion [25]. We have to normalize the partition function by the number of configurations of the absorber given by $\mathcal{Z}_{*m0}$ and by the nth power of the first moment [see Eq. (1)]. For large R on the microscopic scale ℓ the moments of $\rho(r_0)$ at point r_0 in the vicinity of the core of the star scale like

$$\frac{\langle\rho(r_0)^n\rangle}{\langle\rho(r_0)\rangle^n}=\mathcal{Z}_{*mn}/\mathcal{Z}_{*m0}(\mathcal{Z}_{*m1}/\mathcal{Z}_{*m0})^{-n}\sim\left(\frac{R}{\ell}\right)^{-\tau_{mn}}, \quad (28)$$

where $R=S^{\nu}_{a\leq m}$ and the exponents τ_{mn} are given as

$$\tau_{mn}=-\eta_{mn}+n\,\eta_{m1}-(n-1)\,\eta_{m0}. \quad (29)$$

Here, ν is the correlation length critical exponent of the walks: $\nu=1/2$ for random walks and $\nu\simeq 0.588$ for self-avoiding walks at $d=3$. For the fixed point G we have $\eta^G_{m0}=0$ and $\nu=1/2$ for all walks. The scaling near a RW star is then described by inserting the value of η^G, while the scaling near a SAW star is obtained by inserting η^U. In previous work [9,10], we obtained the exponents η^G_{mn}, η^U_{mn} in third order of perturbation theory.

IV. MULTIFRACTAL SPECTRUM

A widely used characterization for the MF spectrum is the so called spectral function $f_m(\alpha)$ [2]. To obtain this function for the absorption process on the center of a star with m legs we analytically continue the set of exponents τ_{mn} in the variable n and calculate the following Legendre transform:

$$f_m(\alpha_{mn})=-\tau_{mn}+n\alpha_{mn}+D_m \quad \text{with} \quad \alpha_{mn}=\frac{d\tau_{mn}}{dn}+D_m. \tag{30}$$

Following the standard definition we have included in Eq. (30) the fractal dimension D_m of the absorber. In particular, this gives the maximal value of the spectral function $f_m(\alpha_{mn})$ to be equal to the dimension D_m. An absorbing chain is described by the case $m=2$, where $D_2=2$ if the chain is a random walk and $D_2=1.71$ if it is self-avoiding. In this special case the midpoint of the chain corresponding to the ''core'' of the 2-star is equivalent to any other point along the chain as far as the MF scaling of the moments of concentration is concerned. This excludes the end point regions that are described by the $m=1$ case. D_2 thus corresponds to the fractal dimension of this set of equivalent points. There appears to be no natural generalization of D_m to arbitrary m where only the moments of concentration near a special point of the absorbing structure (the core of the star) are of interest. In any case D_m only shifts the curve of the spectral function $f(\alpha)$ in the f-α plane by a constant offset. In our presentation we have chosen this offset in such a way that for all m the maximal point of the spectral curve coincides with that of the case $m=2$. This corresponds to the fact that the fractal dimension of a polymer star is equal to that of a linear chain.

In the $m=2$ case the moments of concentration may also be calculated by averaging over all the sites along the chain. For a very long chain one may expect this average to be equivalent to an average of the moments of concentration only at the midpoint site using an ensemble average over all configurations of the chain [7]. The site average has been the original approach to multifractality also due to its easier application in MC simulations. For general m, new interesting behavior occurs only in the vicinity of the core of the star. Only an ensemble average may thus be used to define the moments of concentration near this point. This approach is used here for all values of m. Note that some features of MF spectra that are defined using site averages do not hold for those based on ensemble averages. This will be discussed below.

To obtain the expressions for the spectral function we use the perturbation expansions for the η exponents given to third order both in massless and massive renormalization [9,10]. These exponents are available both in terms of ε-expansion and pseudo-ε-expansion series. The first corresponds to collecting perturbation theory terms of the same power of $\varepsilon=4-d$. In the pseudo-ε-expansion series [29] each power of the pseudo-ε parameter τ collects the contributions from the dimension-dependent loop integrals of the same order. In the final results the limit $\tau=1$ is taken. Starting from the relations for τ_{mn}, Eq. (29), and the spectral function (30) some algebra results in the corresponding expansions for the MF spectra for absorption on stars of random walks (RWs) and self avoiding walks (SAWs):

$$\alpha^{\mathrm{RW}}_{mn}(\varepsilon)=-m(2n-1)\varepsilon^2/8+m[4mn+6n\zeta(3)-12n+3n^2+5-2m-3\zeta(3)]\varepsilon^3/16, \tag{31}$$

$$f^{\mathrm{RW}}_{m}(\varepsilon)=-mn^2\varepsilon^2/8+mn^2[-6+2n+2m+3\zeta(3)]\varepsilon^3/16, \tag{32}$$

$$\alpha^{\mathrm{SAW}}_{mn}(\varepsilon)=-9m(2n-1)\varepsilon^2/128+3m[168mn+54n^2+157+180n\zeta(3)-350n-84m-90\zeta(3)]\varepsilon^3/2048, \tag{33}$$

$$f^{\mathrm{SAW}}_{m}(\varepsilon)=-9mn^2\varepsilon^2/128+3mn^2[-175+36n+84m+90\zeta(3)]\varepsilon^3/2048, \tag{34}$$

$$\alpha^{\mathrm{RW}}_{mn}(\tau)=-m\varepsilon(1+4ni_1-2i_1-2n)\tau^2/4+\alpha^{\mathrm{RW}}_{3\text{-loop}}\tau^3, \tag{35}$$

$$f^{\mathrm{RW}}_{m}(\tau)=-\varepsilon mn^2(-1+2i_1)\tau^2/4+f^{\mathrm{RW}}_{3\text{-loop}}\tau^3, \tag{36}$$

$$\alpha^{\mathrm{SAW}}_{mn}(\tau)=-9m\varepsilon(1-2n+4ni_1-2i_1)\tau^2/64+\alpha^{\mathrm{SAW}}_{3\text{-loop}}\tau^3, \tag{37}$$

$$f^{\mathrm{SAW}}_{m}(\tau)=-9\varepsilon mn^2(-1+2i_1)\tau^2/64+f^{\mathrm{SAW}}_{3\text{-loop}}\tau^3. \tag{38}$$

Here $\zeta(3)\simeq 1.202$ is the Riemann zeta function, i_1, i_2 are the two-loop integrals depending on the space dimension d: at $d=3$ $i_1=2/3$, $i_2=-2/27$. The explicit form of the three-loop contributions in Eqs. (35)–(38) is given in Appendix A.

V. RESUMMATION AND RESULTS

As is well known, the series of type (31)–(38), as they occur in field theory appear to be of an asymptotic nature with zero radius of convergence and are of limited use without resummation (see, e.g., [30]). However, knowing the asymptotic behavior of the series as derived from the RG theory, we may evaluate these asymptotic series. To this end several procedures are available, differing in the amount of information that is used to control the convergence. We extract this additional information for the case of our Lagrangian (13) from [18,31]. We expect the following behavior of the kth order perturbation theory term A_k for any of the above quantities:

$$A_k\sim k!\,k^b(-a)^k. \tag{39}$$

The constant a for the ε expansion of Lagrangian ϕ^4 field theory with one coupling was derived in [31]: $a=3/8$. For the unsymmetric fixed point U, where two different cou-

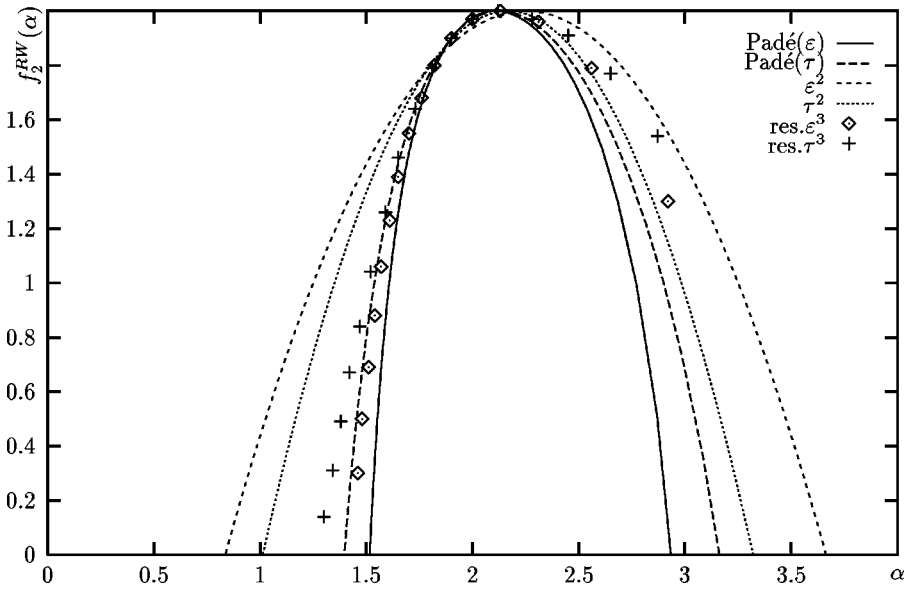

FIG. 2. MF spectrum for diffusion near an absorbing RW. Comparison of different approximation and resummation schemes.

plings are present, the value $a=27/64$ has been proposed [18]. We assume here that the same properties also hold for the pseudo-ε expansion in terms of τ. With the above information at hand one can make use of the Borel summation technique improved by the conformal mapping procedure that has served as a powerful tool in field theory calculations (see [30], for example). We present some details of the resummation procedure in Appendix B.

The results of the resummation of the series for the spectral functions are presented in Figs 2–6. Each point marked by a symbol corresponds to the resummation of both $f_m(\alpha_{mn})$ and α_{mn} for a given pair (m,n), where we used a half-integer spacing for the values of n. Note that the right wings of the curves correspond to negative $n<0$. In this region reliable resummations were feasible only for sufficiently large m. We have only included resummations that were successful in minimizing the deviation between the second and third order resummed values as described in Appendix B.

In Fig. 2 we study the effects of different RG and resummation procedures for $f_2^{\mathrm{RW}}(\alpha)$ in $d=3$ dimensions. The most simple approach is to directly insert $\varepsilon=1$ into the ε expansion, and for the τ expansion to use $\tau=1$ and the $d=3$-dimensional values for the integrals. As can be seen from the curves, this will work only for the series truncated at second order and for small n, i.e., near the maximum of $f_2(\alpha)$ at $n=0$. In addition we have performed an analytical continuation of our series using [2/1] Padé approximants for the series truncated at third order. The symmetry of the Padé approximant holds only in the region shown and may be an artifact of the method. On the left wing, where it coincides with the resummed results, the Padé approximant gives a continuation that is compatible with the estimation for the minimal α value $\alpha_{\min}=d-2$ [7]. The Padé result is $\alpha_{\min}(\varepsilon)=1.333$, $\alpha_{\min}(\tau)=1.017$ for the RW absorber and $\alpha_{\min}(\varepsilon)=1.250$, $\alpha_{\min}(\tau)=1.013$ for the SAW absorber, which is calculated here only from third order perturbation theory. The Padé approximant, while already significantly improving the convergence of the results, introduces some apparently artificial singularities. Moreover, it does not make full use of the known asymptotics for the ε expansions. We have therefore chosen a more sophisticated method of resummation that has

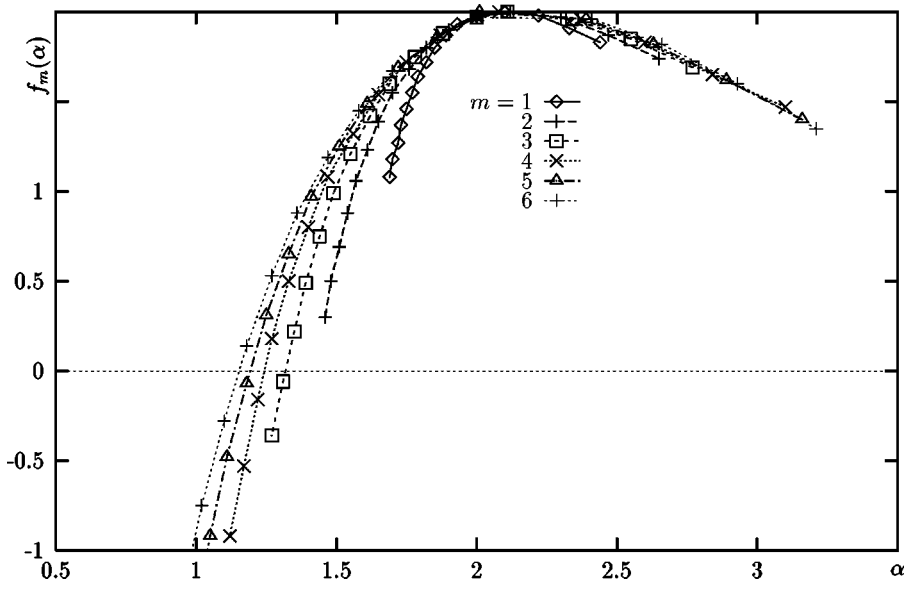

FIG. 3. MF spectra for diffusion near an absorbing RW star (ε expansion).

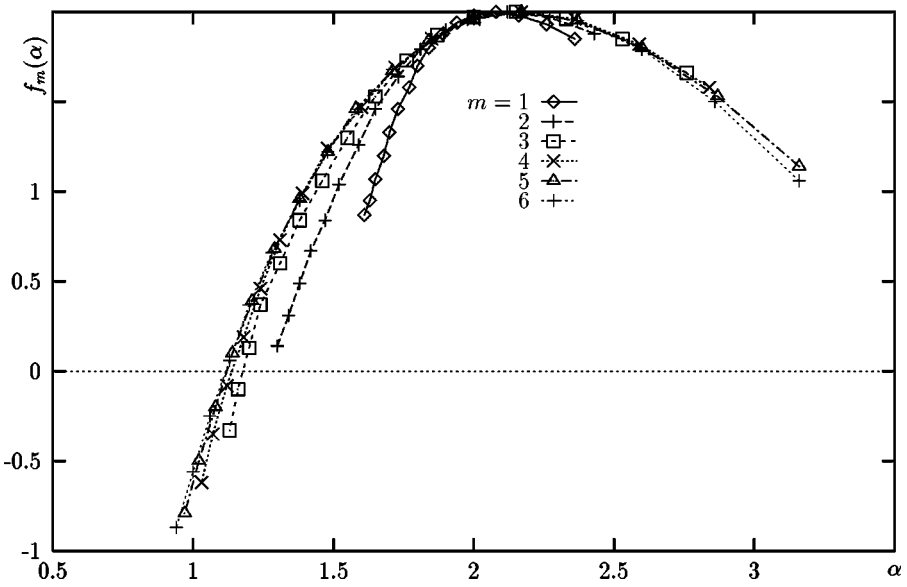

FIG. 4. MF spectra for diffusion near an absorbing RW star (τ expansion).

proven to reproduce reliable data in many field-theoretic applications [28,30]. The results of these resummations are again shown for both RG approaches. Note that though the results obtained for α_n and $f_2(\alpha_n)$ for a specific value of n differ in both approaches, the same curve $f_2(\alpha)$ is described with better coincidence for the left wing of the curves, corresponding to positive n.

Figures 3–6 present the resummed MF spectra $f_m(\alpha)$ of Brownian motion near general m-leg polymer stars in $d=3$ dimensions. The family of curves $f_m(\alpha)$ appears to approach some limiting envelope for increasing m in all cases. This behavior is more pronounced in the case of Brownian motion near an absorbing SAW star. This provides evidence that the MF spectrum catches rather general properties of the phenomena under consideration. Here, for the absorption of diffusing particles on a polymer star the spectrum only slightly varies with the number of legs m of the star, even in the vicinity of the core of the star. Only the absorption on an end point ($m=1$) proves to be an exception.

The behavior of the maximum of the spectra may also be studied in terms of the series expansion. The original position of the maximum is given by its α coordinate in ε expansion in the following form:

$$\alpha_{m,0} = \eta_{m,1} - \eta'_{m,0}, \tag{40}$$

$$\alpha_{\max}^{\mathrm{RW}} = m\varepsilon/8 + \cdots, \tag{41}$$

$$\alpha_{\max}^{\mathrm{SAW}} = m(1-m)\varepsilon/8 + \cdots. \tag{42}$$

For $m>1$ the position of the SAW maximum is shifted in a direction opposite that of the RW maximum. In the ε expansion we find for the curvature at the maximum,

$$1/f''_m(\alpha) = -\eta''_{m,0}, \tag{43}$$

$$1/f''^{\mathrm{RW}}_m(\alpha) = -m\varepsilon^2/4\{1-\varepsilon/2[2m-6+3\zeta(3)]\} + \cdots, \tag{44}$$

$$1/f''^{\mathrm{SAW}}_m(\alpha) = -9m\varepsilon^2/64 + \cdots. \tag{45}$$

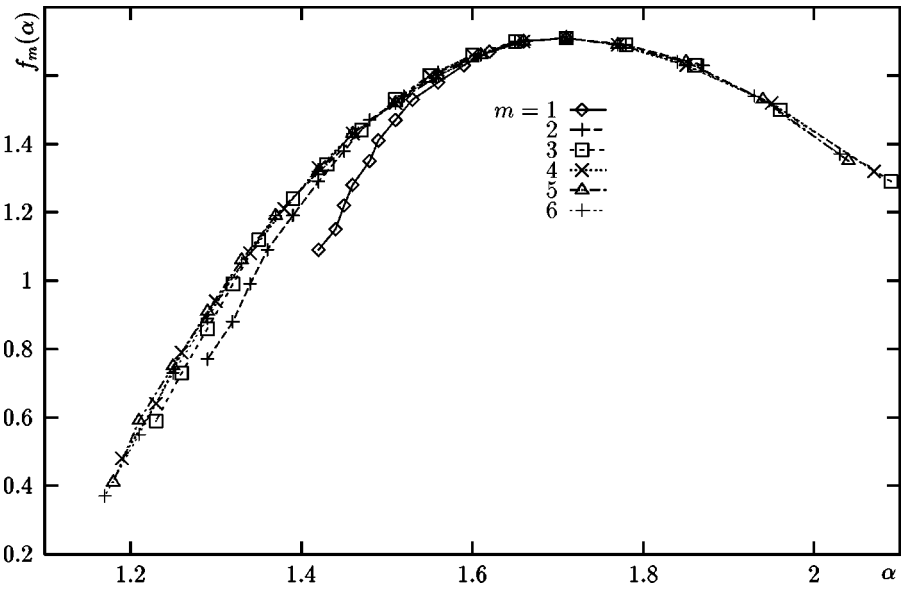

FIG. 5. MF spectra for diffusion near an absorbing SAW star (ε expansion).

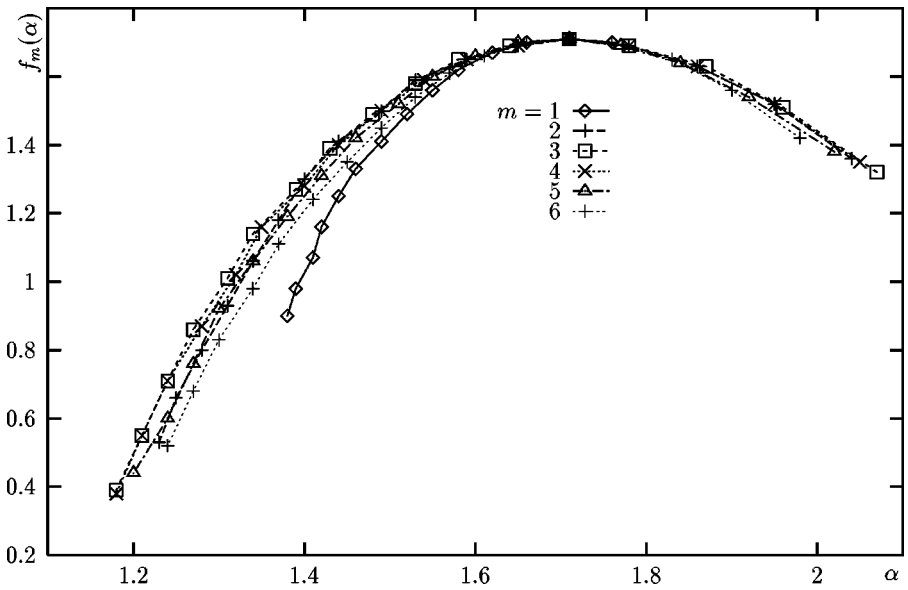

FIG. 6. MF spectra for diffusion near an absorbing SAW star (τ expansion).

Here, we use the notations $f'_m(\alpha)=d/d\alpha f_m(\alpha_{m,n})|_{n=0}$ and $\eta'_{m,n}=d/dn\,\eta_{m,n}|_{n=0}$, and correspondingly for higher derivatives. As can be seen also in the plots, the radius $\mathcal{R}_m \sim 1/f''_m(\alpha)$ of the curvature increases with m for both the RW and SAW star. Some asymmetry is also present in the plots. It may be more explicitly extracted from the series by considering

$$f''_m(\alpha)/f'''_m(\alpha)=(\eta''_{m,0})^2/\eta'''_{0,m}, \tag{46}$$

$$f''^{\mathrm{RW}}_m(\alpha)/f'''^{\mathrm{RW}}_m(\alpha)=m\varepsilon/12[1-\varepsilon(2m-6+3\zeta(3))]+\cdots, \tag{47}$$

$$f''^{\mathrm{SAW}}_m(\alpha)/f'''^{\mathrm{SAW}}_m(\alpha)=m\varepsilon/16[1-\varepsilon(7m/2-175/24+15/64\zeta(3))]+\cdots. \tag{48}$$

This shows that the asymmetry at the maximum decreases slightly with m. The plots seem to indicate that it approaches some limiting value.

From the plots we present here, in general one may deduce that the series for the MF spectra for diffusion near an absorbing polymer star possess stable resummations and that the shape of the resulting curves is robust against the change of the number of legs m of the polymer star, while a limiting curve seems to be approached with increasing m.

VI. CONCLUSIONS

The present work represents extended results on the MF behavior of Brownian motion in the vicinity of an absorbing polymer structure. We extend the ideas of Cates and Witten [7] to map this problem to a problem of interacting walks. The former authors used a Fixman expansion technique to extract the exponents governing the MF scaling. This approach is equivalent to a direct renormalization method and unique to dimensional renormalization with ε expansion. The Fixman expansion assumes without proof the renormalizability of the quantities corresponding to higher moments of the Laplacian field of Brownian motion. Here, we map the problem to an $O(M)$-symmetric ϕ^4-field theory relating the above quantities to corresponding composite operators for which renormalizability has been proven [15]. Furthermore, the scaling exponents of these operators have been calculated in our previous work [9,10]. There, it has been shown that the resulting spectra of scaling dimensions of these operators display the convexity properties that are necessarily found for multifractal scaling but unusual for power of field operators in field theory [6,8].

The extensive RG study in [9,10] with three-loop results allows us here to consider the general case of Brownian motion near the core of a star polymer with m legs. The higher order calculation was improved by resummation to give reliable estimates for the families of MF spectra describing the multiscaling of Brownian motion near absorbing RW or SAW stars.

Our results have proven to be equally stable under the change of the general RG treatment. We applied two complementary RG approaches, the dimensional renormalization with successive ε expansion as well as massive renormalization at fixed space dimensionalities. The resummation in particular enabled us to extend the region over which the curves for the MF spectra coincide in both RG approaches, reflecting the stability of the scheme of calculations. Our plotted results (Figs. 3–6) indicate some independence of the spectral function from the number of legs of the absorbing polymer star. For a higher number of legs the spectrum seems to approach a limiting curve. The MF spectra calculated here show most of the common features shared by spectral functions that describe a variety of MF phenomena. Let us note, however, that unlike the common definition of the underlying scaling exponents based on site averages we rely here on ensemble averages for the moments of the Laplacian field of Brownian motion. We average over the configurational ensembles of absorbing polymer stars. Only for the case of $m=2$ legs of the absorbing star, could a site average definition also be used. As has been noted also in [7,32] the ensemble average leads to the possibility of negative values of the spectral function (see Figs. 2–6). Furthermore, the fractal dimension is not defined for the core of a polymer star.

Experimentally such absorbing systems are realized in diffusion controlled reactions with traps or reaction sites at-

tached to polymer chains. This is described by the irreversible reaction $A+B\to 0$ with freely diffusing particles A and traps B attached along the polymer chain [21]. The higher nth moments of the field at some special trap might then describe the reaction rate for $A^n+C\to 0$. If C is located at the core of a polymer star, this system realizes all aspects of our study. While extensive MC studies exist for many problems in the field of DLA, we hope that our detailed calculations might initiate also an MC approach to the present problem. The current study also allows an extrapolation to $d=2$ dimensions. However, one should not expect to find results of a pure two-dimensional approach, due to topological restrictions in $d=2$ that are not present in the perturbative ε expansion [33].

While standard in field-theoretical studies of critical phenomena, the resummation technique, to our knowledge, has not been applied in the theory of multifractals. We hope that our attempt will attract attention to this possibility, in the context of other problems that arise in the theory of multifractal measures.

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APPENDIX A: THREE-LOOP CONTRIBUTIONS TO THE MULTIFRACTAL SPECTRUM

Here, we collect the three-loop contributions of the expressions for the Hölder exponents α_{mn} [Eqs. (35) and (37)] and the spectral functions $f_m(\alpha_{mn})$ [Eqs. (36) and (38)] obtained in the pseudo-ε-expansion scheme. The expressions read

$$\alpha^{\mathrm{RW}}_{3\text{-loop}}=-\frac{m\varepsilon}{8}(3i_6m-3n^2+3m-6mn+20i_1-6i_4+3i_5-3i_6+3i_7-16{i_1}^2-46ni_1-5+6i_4m+3i_5m+12n-9i_4n^2-12i_4mn+9n^2i_1+32n{i_1}^2-6i_7n-6i_5mn-6i_6mn+6i_6n+18i_4n-12mi_1+24nmi_1-6i_5n), \tag{A1}$$

$$f^{\mathrm{RW}}_{3\text{-loop}}=-\frac{\varepsilon mn^2}{8}(6-2n-6i_4n+9i_4-3i_5m-6i_4m-3i_6m-3i_7-3i_5+12mi_1+16{i_1}^2-23i_1+6ni_1-3m+3i_6), \tag{A2}$$

$$\alpha^{\mathrm{SAW}}_{3\text{-loop}}=-\frac{3m\varepsilon}{512}(-86-718ni_1-198mi_1+36i_6m+126i_4m+36i_5m-27n^2+54m-108mn+332i_1+4i_2-108i_4+36i_5-36i_6+45i_7-72i_5n+72i_6n-90i_7n+496n{i_1}^2+81n^2i_1-72i_5mn-81i_4n^2-252i_4mn-72i_6mn-8i_2i_1+16ni_2i_1-8ni_2+396nmi_1-248{i_1}^2+190n+270i_4n), \tag{A3}$$

$$f^{\mathrm{SAW}}_{3\text{loop}}=-\frac{3\varepsilon mn^2}{512}(95-18n-54i_4n+135i_4-36i_5m-126i_4m-36i_6m-45i_7-36i_5+198mi_1+248i_1^2-359i_1+54ni_1-54m+36i_6-4i_2+8i_2i_1). \tag{A4}$$

The numerical values of the two-loop (i_1, i_2) and three-loop i_3–i_8 integrals at $d=3$ equal [34]: $i_1=2/3$, $i_2=-2/27$, $i_3=-0.037\,682\,072\,5$, $i_4=0.383\,576\,096\,6$, $i_5=0.519\,431\,241\,3$, $i_6=1/2$, $i_7=0.173\,900\,610\,7$, $i_8=-0.094\,651\,431\,9$.

APPENDIX B: RESUMMATION PROCEDURE

Here, we briefly describe the resummation method for the asymptotic series that we applied in our calculations [30,31,35]. The starting point is the expansion for the function of interest:

$$\beta(\varepsilon)=\sum_k A_k\varepsilon^k. \tag{B1}$$

The coefficients A_k are supposed to possess the following behavior:

$$A_k=ck^{b_0}(-a)^k k![1+O(1/k)],\quad k\to\infty \tag{B2}$$

with known values of constants c, b_0, a. The property (B2) indicates the Borel summability of the series (B1). The Borel resummation procedure takes into account the asymptotic behavior of the coefficients and maps the asymptotic series to a convergent one with the same asymptotic limit. The procedure is as follows. For the series (B1) we define a Borel-Leroy transform $f^B(\varepsilon)$ by

$$f^B(\varepsilon)=\sum_j \frac{f^{(j)}\varepsilon^j}{\Gamma(j+b+1)}, \tag{B3}$$

with the Euler gamma function $\Gamma(x)$ and a fit parameter b. Then the initial series may be regained from

$$f^{\mathrm{res}}(\varepsilon)=\int_0^\infty dt t^b e^{-t} f^B(\varepsilon t). \tag{B4}$$

Assuming the behavior of the high order terms (B2), one concludes that the singularity of the transformed series $f^B(\varepsilon)$ closest to the origin is located at the point $(-1/a)$. Conformally mapping the ε plane onto a disk of radius 1 while leaving the origin invariant,

$$w=\frac{(1+a\varepsilon)^{1/2}-1}{(1+a\varepsilon)^{1/2}+1},\quad \varepsilon=\frac{4}{a}\frac{w}{(1-w)^2},$$

and substituting this into $f^B(\varepsilon)$, and reexpanding in w, we receive a series defined on the disk with radius 1 in the w

plane. This series is then resubstituted into Eq. (B4). In order to weaken a possible singularity in the w plane, the corresponding expression is multiplied by $(1-w)^{\alpha}$ and thus one more parameter α is introduced [35]. In the resummation procedure the value of a is taken from the known large-order behavior [18,31] of the ε-expansion series, while α was chosen in our calculations as a fit parameter defined by the condition of minimal difference between resummed second order and third order results. The resummation procedure was seen to be quite insensitive to the parameter b introduced by the Borel-Leroy transformation (B3). The above procedure was applied to both the ε- and pseudo-ε-expansion series.

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